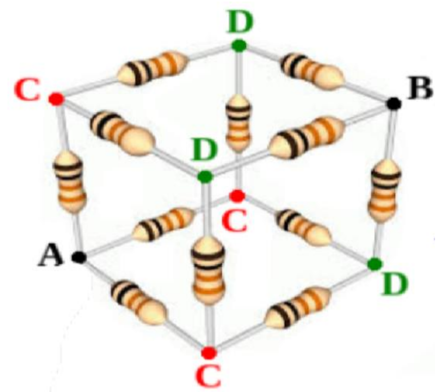
A Comprehensive Analysis of a Cubic Electric Circuit

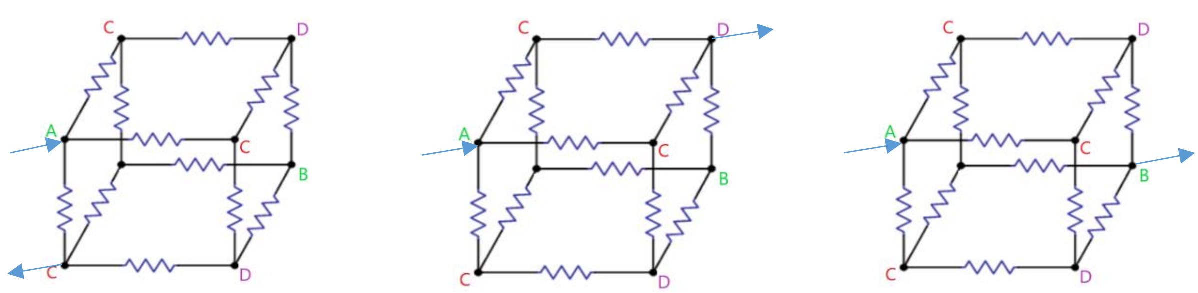
A “cubic circuit” is a 3D arrangement of 12 equal resistors “R” that are connected to form a cube. Two of the vertices of this cube are connected to a power supply. In this analysis, we will calculate the equivalent resistance RT for three different input gates.



Introduction

As shown in the circuit diagram, one of the two input terminals can always be the vertex “A” and the other one can be C, D, or B. Therefore, the cubic circuit can be classified into three categories as illustrated:

1. Battery connected to one edge of the cube (A-C)
2. Battery connected to a diagonal of one face of the cube (A-D)
3. Battery connected to the diagonal of the cube (A-B)

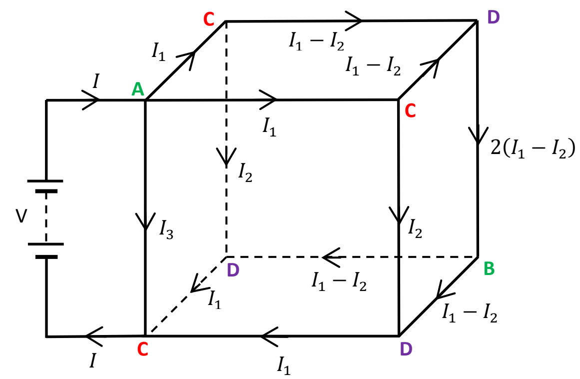


1. **2.** **3.**

As it will be shown, the circuit has symmetry in each category, so the power supply can be connected in the same way to any edge, face diagonal or cube diagonal. All the resistors have the same value “R”. Let RT be the equivalent resistance in each case.

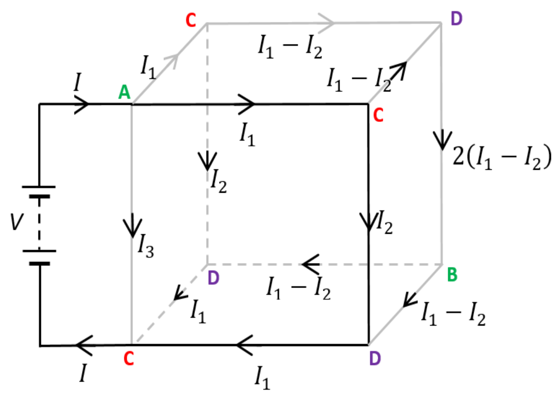
Solutions:

1. **Input output at One Edge (side):**
   (Across the Resistor AC).

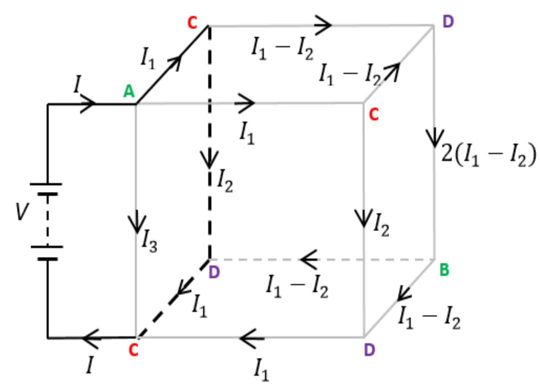


In this case, the current can take two different, but similar paths illustrated in the following diagrams. Figures 1 and 2 are two identical loops: ACDC

1. Front face: ACDC

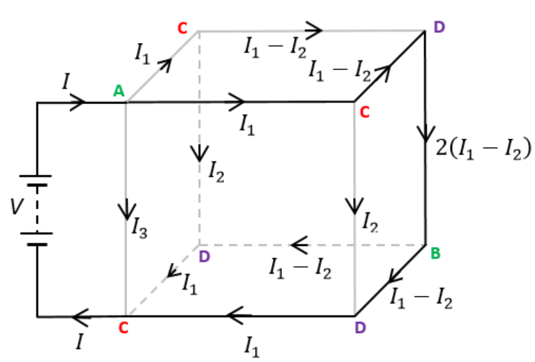


2. Left face: ACDC

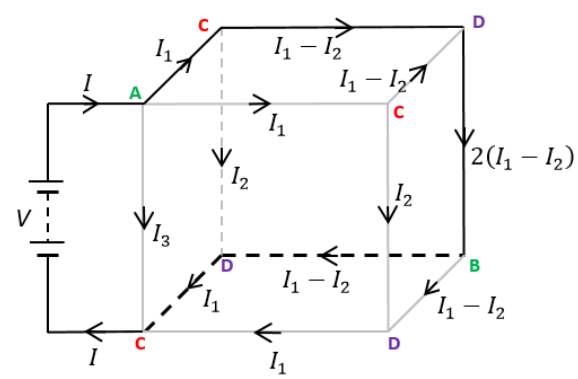


Figures 3 and 4 are also two identical loops; ACDBDC:

3. Front face then back: ACDBDC



4. Back then back face: ACDBDC



In addition the 2 longer paths "ACDCDC", contribute the same way in configurations "1 and 3" also in "2 and 4", therefore we can say calculation for the two main paths are all similar due to symmetry. So we proceed and treat them the same way.
According to those diagrams, we apply Kirschahf's current law (KCL) for the junction A:

$$I = 2I_1 + I_3 \qquad (1)$$

Recall: $I = V/R_T$ with: $I_3 = V/R$

Then write Kirschahf's voltage law (KVL) for loops 1 or 2:

$$2I_1 R + I_2 R = V \qquad (2)$$

And KVL for loops 3 or 4:

$$2I_1 R + 4(I_1 - I_2) R = V \qquad (3)$$

Now solve the system of 3 equations: 1, 2, and 3.

Use (3): $6I_1 R + 4 I_2 R = V$ Then: $6I_1 + 4I_2 = V/R \qquad (5)$

By simplifying (2) we get: $2I_1 + I_2 = V/R \qquad (6)$

Now solve the system of two equations (5) and (6):

$$14I_1 = 5V/R$$

Therefore:

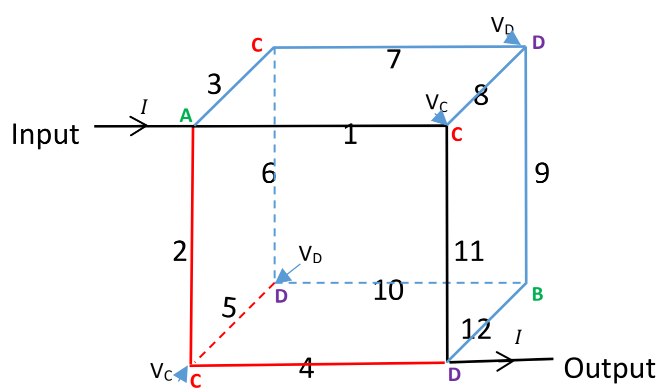
$I_1 = \frac{5}{14}\left(\frac{V}{R}\right)$, $I_2 = \frac{4}{14}\left(\frac{V}{R}\right)$ and from (1): $I = \frac{10}{14}\left(\frac{V}{R}\right) + \left(\frac{V}{R}\right) = \frac{24}{14}\left(\frac{V}{R}\right) = \frac{12}{7}\left(\frac{V}{R}\right)$

Considering: $I = \frac{V}{R_T}$ and: $I = \frac{12}{7}\left(\frac{V}{R}\right)$

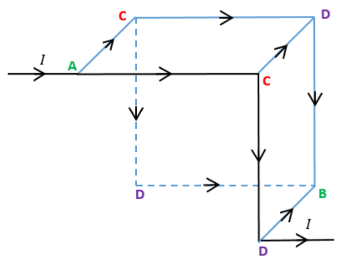
$$\boxed{R_T = \frac{7}{12} R}$$

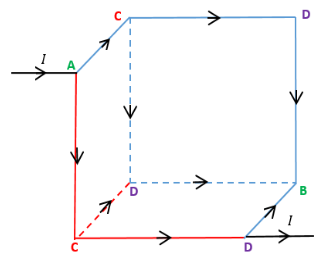
2. Input and output are in the diagonal of one face

Vertex "A" is the input, and the output is one of the D vertices, on the same face:

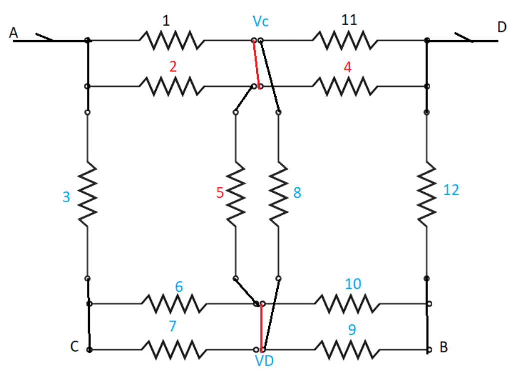


The simplification in this case is quite different from the other two cases. As illustrated in the diagram, the current can take two different, but similar paths.

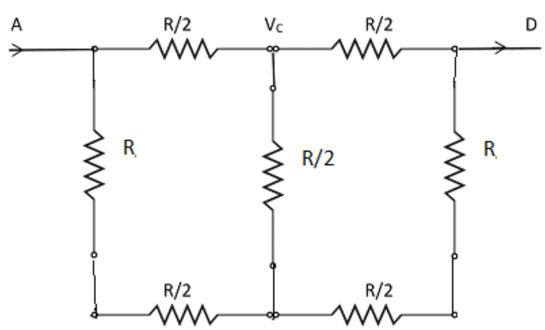




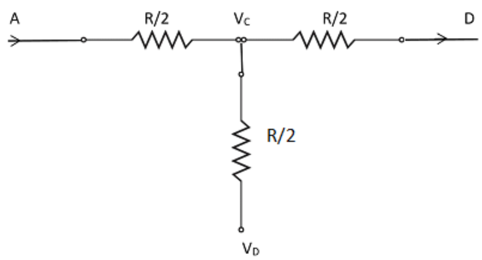
Due to symmetry, the junction “C” of resistors (2, 4, 5) and resistors (1, 8, 11) have the same potential (V_C). So we can superimpose them like they are connected. In similar way, junctions “D” (7, 8, 9) and (5, 6, 10) have the same potential (V_D).
The 2 dimensional presentation of the equivalent circuit will be set on the following diagram:

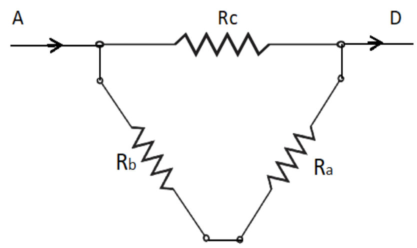


As can be seen in the diagram above the 2 points C, in the same face with the same potential are connected with a red line, same way for points D, now we need to simplify parallel resistors in the circuit. Given all the resistors are equal to R:



Now apply star (Y) to triangle (Δ) transformation for the 3 central resistors R/2 on top:

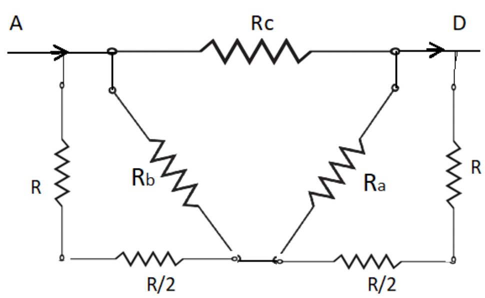


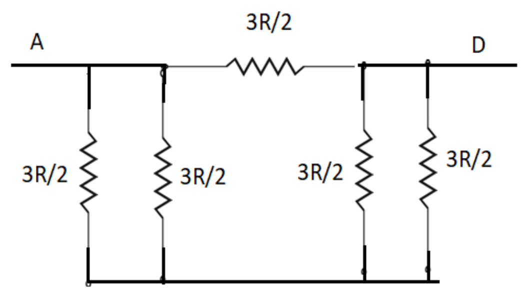


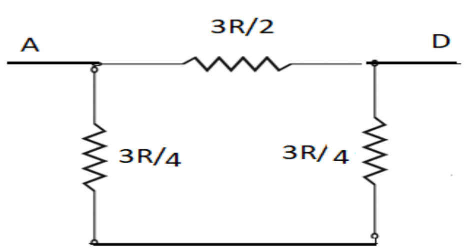
The star circuit: $R_1 = R_2 = R_3 = R/2$. The triangle configuration we will get:

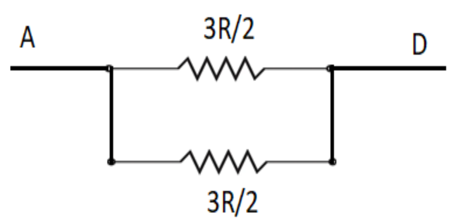
$$R_a = R_b = R_c = \frac{R1.R2+R2.R3+R1.R3}{R1} = \frac{3(\frac{R}{2})(\frac{R}{2})}{(\frac{R}{2})} = 3R/2$$

Next:





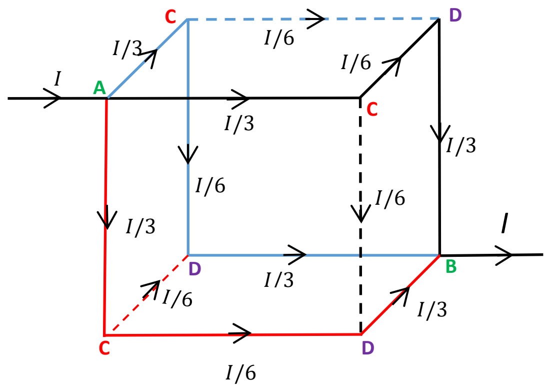




Therefore the total resistance is: $\boxed{R_T = \frac{3}{4}R}$

3. Input – output are the cube diagonal 3D:

In the following illustration, there are 3 paths ACDB; black, red, and blue; due to symmetry, in any of these three paths the distribution of current “I” will be as follow:

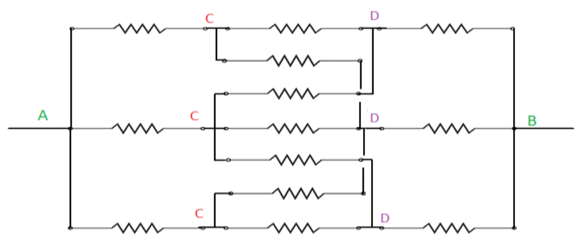


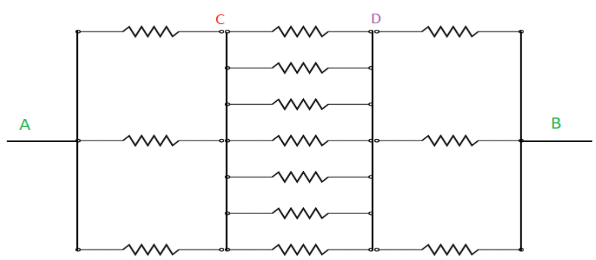
By applying Ohm's Law for the sum of the voltage in each path (series circuit):

$$V = R\,(I/3) + R\,(I/6) + R(I/3) = 5/6\ R.I$$

Therefore: $R_T = V/I = 5R/6$

Alternative solution: the total equivalent resistance:





We know that all the resistances are equal R, so the simplified circuit will give us:

Therefore: $\boxed{R_T = 5/6R}$